

SOLVING POLYNOMIAL EQUATIONS USING THE FACTOR THEOREM

For all the polynomial equations below (questions 1 – 3):

- Use the Factor Theorem to find one factor
- Divide the polynomial by the factor to get a quadratic
- Factorise the quadratic
- Rewrite the polynomial equation in factorised form
- Solve the factorised equation.

Questions

1) Solve $2x^3 - 3x^2 - 23x + 12 = 0$

2) Solve $3x^3 + 2x^2 - 7x + 2 = 0$

3) Solve $6x^3 + x^2 - 11x - 6 = 0$

4) $f(x) = x^3 + kx^2 - 4x + 12$ and $(x - 3)$ is a factor of $f(x)$. Calculate the value of k .

Solutions

1) Solve $2x^3 - 3x^2 - 23x + 12 = 0$

a) $f(-3) = 0 \Rightarrow (x + 3)$ is a factor.

b)

$$\begin{array}{r} 2x^2 - 9x + 4 \\ x+3 \overline{) 2x^3 - 3x^2 - 23x + 12} \\ \underline{2x^3 + 6x^2} \\ -9x^2 - 23x \\ \underline{-9x^2 - 27x} \\ 4x + 12 \\ \underline{4x + 12} \\ 0 \end{array}$$

c) $2x^2 - 9x + 4 = (2x - 1)(x - 4)$

d) $(x + 3)(2x - 1)(x - 4) = 0$
 \Rightarrow solutions are $x = -3, x = \frac{1}{2}, \text{ and } x = 4$.

2) Solve $3x^3 + 2x^2 - 7x + 2 = 0$

a) $f(-2) = 0 \Rightarrow (x + 2)$ is a factor.

b)

$$\begin{array}{r} 3x^2 - 4x + 1 \\ x+2 \overline{) 3x^3 + 2x^2 - 7x + 2} \\ \underline{3x^3 + 6x^2} \\ -4x^2 - 7x \\ \underline{-4x^2 - 8x} \\ x + 2 \\ \underline{x + 2} \\ 0 \end{array}$$

c) $3x^2 - 4x + 1 = (3x - 1)(x - 1)$

d) $(x + 2)(3x - 1)(x - 1) = 0$
 \Rightarrow solutions are $x = -2, x = \frac{1}{3}, \text{ and } x = 1$.

3) Solve $6x^3 + x^2 - 11x - 6 = 0$

a) $f(-1) = 0 \Rightarrow (x + 1)$ is a factor.

b)

$$\begin{array}{r}
 6x^2 - 5x - 6 \\
 x+1 \overline{) 6x^3 + x^2 - 11x - 6} \\
 \underline{6x^3 + 6x^2} \\
 -5x^2 - 11x \\
 \underline{-5x^2 - 5x} \\
 -6x - 6 \\
 \underline{-6x - 6} \\
 0
 \end{array}$$

c) $6x^2 - 5x - 6 = (3x + 2)(2x - 3)$

d) $(x + 1)(3x + 2)(2x - 3) = 0$

\Rightarrow solutions are $x = -1, x = -\frac{2}{3},$ and $x = \frac{3}{2}.$

4) $f(x) = x^3 + kx^2 - 4x + 12$ and $(x - 3)$ is a factor of $f(x)$. Calculate the value of k .

Since $(x - 3)$ is a factor, then $f(3) = 0$:

$$\begin{aligned}
 3^3 + k(3)^2 - 4(3) + 12 &= 0 \\
 27 + 9k - 12 + 12 &= 0 \\
 27 + 9k &= 0 \\
 k &= -3
 \end{aligned}$$